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PSEUDO-COMPACT SUBSETS OF INFINITE-DIMENSIONAL MANIFOLDS

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Introduction

In the last few years there has been considerable progress in the theory of Hilbert manifolds, and the answers to almost all questions turned out to be as simple as they possibly could be. In particular, as Eells and Elworthy have shown, every Hilbert manifold is diffeomorphic to an open subset of Hilbert space; a result of Kuiper and Burghelea then implies that two Hilbert manifolds are diffeomorphic if they are homotopically equivalent. One of the main reasons for this lack of complications is perhaps that there is nothing built into the definition of a Hilbert manifold which could play the role compact subsets play in finite dimensions. This let Palais to suggest that one should add structure to infinite-dimensional manifolds by specifying what subsets one considers a being "pseudo-compact". He also suggested a definition which is motivated by K. Uhlenbeck's notion of intrinsically bounded subsets of Sobolevmanifolds of sections.

The purpose of this paper is to investigate manifolds with pseudo-compact structure. In particular, we introduce the notion of ϕ-boundaries (which correspond to compactifying boundaries of finite-dimensional manifolds), derive a strong invariant and use it to obtain the complete classification of a large class of manifolds with pseudo-compact structure as well as criteria for the existence of open embeddings. Finally, we compute our invariant for many concrete examples and end up with some deletion theorems for section manifolds which might have some interest in the theory of non-linear elliptic operators.

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1. Pseudo-compacta

Let M be a Banach manifold (i.e., a Hausdorf space locally homeomorphic to open subsets of suitable Banach spaces), and fix a (not necessarily maximal) atlas $\mathscr{A}$ for M.

Definition (*Palais*). A subset K of M is *pseudo-compact* (with respect to $\mathscr{A}$)

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