J. DIFFERENTIAL GEOMETRY
5 (1971) 61-74

THE DIAMETER OF δ-PINCHED MANIFOLDS

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0. Introduction

It is interesting to investigate the manifold structures of a complete riemannian manifold whose sectional curvature is bounded below by a positive constant. As is well known such a riemannian manifold is compact and we may suppose that its sectional curvature K_σ satisfies $0 < \delta \leq K_\sigma \leq 1$ for every plane section σ. Berger proved in [2] and [3] that a complete, simply connected and even dimensional riemannian manifold with $\delta = 1/4$ is homeomorphic to a sphere, or otherwise M is isometric to one of the compact symmetric spaces of rank one. For arbitrary dimensional riemannian manifolds, Klingenberg proved in [8] that a complete and simply connected riemannian manifold with $\delta > 1/4$ is homeomorphic to a sphere. Moreover, Berger claimed in [4] that M is a homology sphere if the diameter $d(M)$ of M satisfies $d(M) > \pi/(2\sqrt{\delta})$ for $0 < \delta \leq 1$.

Since the diameter $d(M)$ of a δ-pinched manifold M plays an important role in the proofs of these interesting results mentioned above, it might be significant to investigate the relationship between the manifold structure of M and its diameter $d(M)$ of a δ-pinched riemannian manifold.

One of our main results obtained in the present paper is:

A connected and complete riemannian manifold with $\delta = 1/4$ is homeomorphic to a sphere if the diameter $d(M)$ of M satisfies $d(M) > \pi$.

For a simply connected riemannian manifold with $\delta = 1/4$, Klingenberg claimed in [9] that the distance $d(p, C(p))$ between any point $p \in M$ and its cut locus $C(p)$ is no less than π, and M is either homeomorphic to a sphere or M is isometric to one of the compact symmetric spaces of rank one. However the proof stated in [9] seems to us to be incomplete[1].

As the main theorem, it will be proved that *a three dimensional, connected, complete and orientable riemannian manifold with $\delta > 1/4$ is isometric to the lens space $L(1, k)$ of constant curvature* 1, *if M has a closed geodesic segment Γ with the length $\mathscr{L}(\Gamma) = 2\pi/k$ and the fundamental group $\pi_1(M)$ of M satisfies $\pi_1(M) = Z_k$, where k is an odd prime.*

Definitions and notations are given in § 1. In § 2, we shall give an estimate

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Communicated by W. P. A. Klingenberg, October 13, 1969.

[1] **Added in Proof.** Recently J. Cheeger proved this theorem completely.