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# ASYMPTOTIC BEHAVIOUR OF NON-PARAMETRIC MINIMAL HYPERSURFACES

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## 1. Introduction

The main result of this paper is Theorem II, which deals with the asymptotic behaviour of non-parametric complete minimal hypersurfaces. In [2], Bombieri, De Giorgi and Giusti showed that hypersurfaces of this type other than hyperplanes exist for dimensions greater than seven. For lower dimensions Theorem II is vacuous because, by theorems of Almgren [1], De Giorgi [6], and Simons [7], the only hypersurfaces of this type are the hyperplanes. The proof of Theorem II relies on Theorem I, which states that the Gauss map of a hypersurface with constant mean curvature is harmonic. A map of Riemannian manifolds is called harmonic if it is an extremal of a certain energy integral which generalizes the classical Dirichlet integral. An extensive study of harmonic maps has been done by Eells and Sampson [3]. If the dimension of a minimal surface is two, then the Gauss map is holomorphic; this is much stronger than harmonic. We expect that the weaker property will still be useful in extending to higher dimensions some of the theorems on 2-dimensional minimal surfaces which are obtained through complex function theory. An example of this method is the proof of Theorem II.

## 2. Harmonic maps

At this point we begin the discussion of Theorem I. A map $f\colon M_1 \to M_2$ of Riemannian manifolds is called harmonic if $f$ is an extremal of the integral

$$E(f) = \int Tr\, f^*g\, dv ,$$

where $Tr\, f^*g$ denotes the trace of the pullback under $f$ of the Riemannian metric $g$ on $M_2$, and $dv$ denotes the volume form on $M_1$. For our purposes, the map in question will be the Gauss map $n\colon M \to S^n$ which sends a point on $M$ into its unit normal vector. Now we are in aposition to state the first theorem.

**Theorem I.** *If $i\colon M \to E^{n+1}$ is an isometric immersion of the n-dimensional*

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