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HIGHER ORDER CONSERVATION LAWS. II

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1. Introduction

Suppose that θ is an exact differential form of degree 1 and h is a vector 1-form. Then θ is a conservation law for $\underline{h}$ if $\underline{h}\theta$ is also exact. The Nijenhuis tensor $[\underline{h}, \underline{h}]$ of a vector 1-form $\underline{h}$ with distinct eigenvalues plays an important role in the study of conservation laws and their generalizations on an analytic manifold. For example, the vanishing of this tensor guarantees the existence of a basis of exact 1-forms $(dv^1, \cdots, dv^n)$ which are also eigenforms of $\underline{h}$. An important consequence of this fact is that a differential from θ of degree p is a higher order conservation law if and only if it has a representation

$$\sum_{i_1<\cdots<i_p} \mathscr{A}_{i_1\cdots i_p}(v^{i_1}, \cdots, v^{i_p}) dv^{i_1} \wedge \cdots \wedge dv^{i_p} .$$

The preceding result is established in *Higher order conservation laws* [5], hereafter referred to as HOCL.

In the present paper higher order conservation laws for vector 1-forms $\underline{h}$ and $\underline{k}$ which commute are studied. The results which are obtained include as special cases certain theorems which appear in [4]. Some of the notation which was established in HOCL is reviewed briefly in § 2 of this paper.

2. Notation and definitions

The ring of germs of analytic functions at some point of an analytic manifold is denoted by A, and the localization of the A-module of differential forms on this manifold is denoted by $\mathscr{E}$. The exterior algebra $\Lambda^*\mathscr{E}$ generated by $\mathscr{E}$ is a direct sum

$$\Lambda^*\mathscr{E} = \Lambda^0\mathscr{E} \oplus \Lambda^1\mathscr{E} \oplus \cdots \oplus \Lambda^n\mathscr{E} ,$$

where $\Lambda^0\mathscr{E} = A$ and $\Lambda^1\mathscr{E} = \mathscr{E}$. An element $\underline{h} \in \operatorname{Hom}_A(\mathscr{E}, \mathscr{E})$ induces homomorphisms

$$\Lambda^p\mathscr{E} \xleftarrow{h^{(q)}} \Lambda^p\mathscr{E} , \qquad 0 \le q \le p ,$$

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