J. DIFFERENTIAL GEOMETRY
4 (1970) 457-467

SUBMANIFOLDS OF CODIMENSION 2 WITH CERTAIN PROPERTIES

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Introduction

H. Liebmann [5] has proved that the only ovaloid with constant mean curvature in a 3-dimensional Euclidean space is a sphere. Various generalizations of this theorem have been obtained recently. Y. Katsurada [1], [2] and K. Yano [9] have generalized this theorem to a hypersurface of a Riemannian manifold admitting an infinitesimal conformal or homothetic transformation. A generalization of the theorem to the case of codimension greater than 1 was first tried by the present author [8] when the enveloping Riemannian manifold is an odd dimensional sphere. In [8], the present author made full use of the natural contact structure on the sphere.

On the other hand Y. Katsurada [3], [4], H. Kōjyo [3], T. Nagai [4] and K. Yano [10] studied this problem when the enveloping manifold admits an infinitesimal conformal transformation. They made full use of the existence of an infinitesimal conformal transformation, and proved that under some conditions the submanifold in consideration is umbilical only with respect to the mean curvature normal. In the present paper the author studies the same problem as that in [3], [4], [10] and proves that under certain conditions the submanifold in consideration is not only umbilical with respect to the mean curvature normal but also is totally umbilical.

In § 1 we recall formulas for the submanifolds of codimension 2 in a Riemannian manifold which will be used in the sequel.

In § 2 we define a certain intrinsic normal vector field and consider some properties of the normal bundle. In § 3 we derive some integral formulas for a compact submanifold of codimension 2 in a Riemannian manifold admitting an infinitesimal conformal transformation. Using these formulas, we establish, in § 4, a certain generalization to the most general form of Liebmann's theorem above stated. In the last § 5 we study submanifolds of codimension 2 of a sphere or a Euclidean space.

1. Submanifolds of codimension 2 in a Riemannian manifold

Let M^n be an n-dimensional orientable differentiable manifold, and ι be an

Received June 3, 1969.