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CURVATURE STRUCTURES AND CONFORMAL TRANSFORMATIONS

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By a curvature structure on a Riemann manifold (M, g) we mean any (1,3) tensorfield which has the algebraic properties of the Riemann curvature tensor. Some examples are given in § 1.

Let $G_2(M)$ denote the Grassmann bundle of 2-planes on M. A curvature structure naturally defines the corresponding sectional curvature, which is a real-valued function on $G_2(M)$. In this memoir we shall show that these sectional curvature functions are of considerable geometrical interest.

Let (M, g), $(\bar{M}, \bar{g})$ be two Riemann manifolds, and let K(resp. $\bar{K}$) be the usual sectional curvature functions canonically defined by g(resp. $\bar{g}$). Call (M, g), $(\bar{M}, \bar{g})$ *isocurved* if there exists a 1-1 onto sectional-curvature-perserving diffeomorphism $f: M \to \bar{M}$, i.e., for every $p \in M$, $\sigma \in G_2(M)_p$, $K(\sigma) = \bar{K}(f_* \sigma)$. The "theorema egregium" or what is essentially the "fundamental theorem of Riemann geometry" asserts that isometric manifolds are isocurved. The basic result of [8] is the converse.

Call (M, g) *nowhere of constant curvature* if there does not exist a nonempty open subset on which $K \equiv$ constant. We have

Theorem A. *Let (M, g), $(\bar{M}, \bar{g})$ be isocurved. Suppose that (M, g) is nowhere of constant curvature and of dimension ≥ 4. Then (M, g), $(\bar{M}, \bar{g})$ are isometric.*

In the following we use the techniques developed in the proof of this theorem. All manifolds in this paper are assumed to be connected; and all manifolds, metrics and maps are assumed to be C^4.

PART I. CURVATURE STRUCTURES

Introduction

In this part, we first develop some generalities on curvature structure. These are applied to two cases: conformal curvature structure which is defined by the conformal curvature tensor, and the Ricci curvature structure which is defined by a certain combination of the Ricci tensor.

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