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CONFORMALLY FLAT RIEMANNIAN MANIFOLDS ADMITTING A ONE-PARAMETER GROUP OF CONFORMAL TRANSFORMATIONS

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Introduction

It is a classical theorem of H. A. Schwarz [4] that a compact Riemann surface of genus greater than 1 has only a finite number of conformal transformations. It is furthermore known that in the case of genus 1, i.e. on a torus, there is no one-parameter group of non-isometric conformal transformations.

For Riemannian manifolds the following is a well-known conjecture:

Conjecture. *If a compact Riemannian n-manifold M, $n > 2$, admits an essential one-parameter group of conformal transformations, then M is conformorphic to a Euclidean sphere S^n.*

Here a one-parameter group and the vector field defined by it are said to be *essential* if there is no Riemannian metric, conformal to the original one, with respect to which the group is a group of isometries, and by "conformorphic" we mean "conformally diffeomorphic". Furthermore, throughout this paper manifolds under consideration are assumed to be connected and of C^∞.

In a previous paper [3], we have proved the following Theorems A and B:

Theorem A. *Let M be a Riemannian n-manifold, $n > 2$, admitting an essential one-parameter graup f_t of conformal transformations. Then*

(i) *f_t has a fixed point,*

(ii) *M is conformorphic to either a Euclidean n-sphere S^n or a once-punctured n-sphere $S^n - \{p_\infty\}$ provided that the vector field defined by f_t has nonvanishing divergence at each of the fixed points of f_t.*

The vector field induced by a one-parameter group of conformal transformations is called a *conformal* vector field, and a fixed point of a one-parameter group is called a *zero* or a *singular point* of the corresponding vector field.

Theorem B. *Let u be an essential conformal vector field on a Euclidean n-sphere S^n. Then u satisfies one of the following two properties:*

(i) *u has exactly one singular point p_0, at which the divergence of u vanishes, and the orbit $f_t(p)$ of u through a point p satisfies*

$$\lim_{t \to \pm\infty} f_t(p) = p_0 \ ,$$

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