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SUBMANIFOLDS OF COSYMPLECTIC MANIFOLDS

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1. Introduction

Recently B. Smyth [6] has classified those complex Einstein hypersurfaces of a Kaehler manifold of constant holomorphic curvature. This paper was followed by the papers of Chern [2], Nomizu and Smyth [4], Kobayashi [3] and others researching this problem. Yano and Ishihara [7] have studied the analogous problem for Sasakian manifolds, i.e., they have studied invariant Einstein (or η-Einstein) submanifolds of codimension 2 of a normal contact manifold of constant curvature. The result of Smyth rests on the fact that the hypersurface is locally symmetric. We show in this paper that a normal contact manifold which is η-Einsteinian but not Einsteinian cannot be locally symmetric. Thus, since an invariant submanifold of codimension 2 in a normal contact manifold is itself a normal contact manifold, the η-Einstein case studied by Yano and Ishihara will not yield to a study similar to that of Smyth.

Let $\tilde{M}$ be a normal contact manifold or a cosymplectic manifold of constant $\tilde{\phi}$-sectional curvature, and M an invariant submanifold of codimension 2. The main purpose of this paper is to study the case where M is η-Einsteinian. In particular, we show that if $\tilde{M}$ is cosymplectic then M is locally symmetric. This suggests that a classification similar to that of Smyth may be obtained in this case.

2. Almost contact manifolds

Let $\tilde{M}$ be a C^∞-manifold and $\tilde{\phi}$ a tensor field of type (1, 1) on $\tilde{M}$ such that

$$\tilde{\phi}^2 = -I + \tilde{\xi} \otimes \tilde{\eta} ,$$

where I is the identity transformation, $\tilde{\xi}$ a vector field, and $\tilde{\eta}$ a 1-form on $\tilde{M}$ satisfying $\tilde{\phi}\tilde{\xi} = \tilde{\eta} \circ \tilde{\phi} = 0$ and $\tilde{\eta}(\tilde{\xi}) = 1$. Then $\tilde{M}$ is said to have an *almost contact structure*. It is known that there is a positive definite Riemannian metric $\tilde{g}$ on $\tilde{M}$ such that $\tilde{g}(\tilde{\phi}X, Y) = -\tilde{g}(X, \tilde{\phi}Y)$ and $\tilde{g}(\tilde{\xi}, \tilde{\xi}) = 1$, where X and Y are vector fields on $\tilde{M}$. Define the tensor $\tilde{\Phi}$ by $\tilde{\Phi}(X, Y) = \tilde{g}(X, \tilde{\phi}Y)$. Then $\tilde{\Phi}$ is a 2-form. If $[\tilde{\phi}, \tilde{\phi}] + d\tilde{\eta} \otimes \tilde{\xi} = 0$, where $[\tilde{\phi}, \tilde{\phi}](X, Y) = \tilde{\phi}^2[X, Y] + [\tilde{\phi}X, \tilde{\phi}Y] - \tilde{\phi}[\tilde{\phi}X, Y] - \tilde{\phi}[X, \tilde{\phi}Y]$, then the almost contact structure is said to be *normal*. If $\tilde{\Phi} = d\tilde{\eta}$, the almost contact structure is a *contact structure*.

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