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THE NUMBER OF BRANCH POINTS OF SURFACES OF BOUNDED MEAN CURVATURE

ERHARD HEINZ & STEFAN HILDEBRANDT

Introduction

Let B be the unit disk $|w|<1$ $(w = u+iv)$, and $\mathfrak{x} = \mathfrak{x}(w) = (x(w), y(w), z(w))$ a vector function of class $C^2(B) \cap C^0(\bar{B})$ satisfying in B the partial differential equations

$$\Delta \mathfrak{x} = 2H(w)(\mathfrak{x}_u \times \mathfrak{x}_v) \,, \tag{0.1}$$

and

$$\mathfrak{x}_u^2 = \mathfrak{x}_v^2 \,, \qquad \mathfrak{x}_u \mathfrak{x}_v = 0 \,, \tag{0.2}$$

where $H = H(w)$ is a bounded function on $\bar{B}$. Furthermore, assume that we have

$$|\mathfrak{x}(w)| \leq 1 \qquad (w \in \bar{B}) \,, \tag{0.3}$$

and

$$A(\mathfrak{x}) = \frac{1}{2} \iint_B (\mathfrak{x}_u^2 + \mathfrak{x}_v^2) du\, dv < +\infty \,, \tag{0.4}$$

and that $\mathfrak{x} = \mathfrak{x}(w)$ maps the unit circle ∂B topologically onto a closed rectifiable Jordan curve $\Gamma^* \subset R^3$. Geometrically speaking, these conditions express the fact that $\mathfrak{x} = \mathfrak{x}(w)$ represents a surface in R^3 of finite area $A(\mathfrak{x})$, contained in the unit ball $|\mathfrak{x}| \leq 1$, which is bounded by Γ^* and whose mean curvature at each regular point coincides with the function $H(w)$.

The various existence proofs for such surfaces available in the literature ([2], [5], [6], [14], [15], and [16] deal with the case, where $H(w)$ is a constant, while [7] treats the general case) leave the question open, whether, for a given curve Γ^*, there always exists a surface of prescribed mean curvature which is free of branch points. While even for minimal surfaces ($H(w) \equiv 0$) this is an unsolved problem, it is nevertheless possible to estimate the total number of these branch points in terms of geometric quantities associated with Γ^*. The

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