J. DIFFERENTIAL GEOMETRY
4 (1970) 207-223

SOME DIFFERENTIAL INVARIANTS OF SUBMANIFOLDS OF EUCLIDEAN SPACE

JAMES H. WHITE

1. Introduction

Let $f\colon M^s \to E^n$ be a C^∞ immersion of an oriented differentiable manifold with or without boundary into Euclidean space of dimension n, p an arbitrary generic (in a sense which will be made clear in § 2) point of E^n, and N a fiber space over M which is mapped in a C^∞ fashion by a function g into E^n. In this paper we prove a number of differential topological and integral geometric formulas relating the intersection number of N with p to the integrals of certain differential invariants of M.

In § 2, we prove the main equation from which all our results follow. In § 3, we consider the case where $f\colon M^{n-1} \to E^n$ is an immersion of a hypersurface and N is a particular submanifold of the normal bundle. Here the intersection number is seen to relate the normal degree of the immersion to the linking number of the immersion with the point p.

In § 4, we consider the simple case of curves in three-space and find new integral formulas for the total curvature and total torsion of a closed space curve. In § 5, we present the general theory in which we introduce, for s odd, integral formulas for new differential invariants generalizing the curvature and torsion of a space curve. For s even we obtain differential topological results relating the Euler classes of certain s-plane bundles to our intersection number. In particular, in § 6 we prove that if $f\colon M^s \to E^{n=s+k}$ is an immersion of an oriented compact manifold M^s and if N is a k-plane bundle over M^s and p a point of E^n, then the intersection number of N with p is the Euler class of the complementary s-plane bundle evaluated on the fundamental class of M^s.

Finally, § 7 deal with manifolds M^s with boundary and gives a new formulation of the Gauss-Bonnet theorem for arbitrary codimension.

In all that follows all manifolds and fiber spaces are to be assumed C^∞ and oriented, and all maps are to be asumed C^3.

The author wishes to thank Professor William Pohl for some very helpful criticisms of this paper.

Communicated by S. S. Chern, June 17, 1969, and, in revised form, September 22, 1969. Research supported in part by National Science Foundation Grant GP-11476.