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THE SPHERICAL TWO-PIECE PROPERTY AND TIGHT SURFACES IN SPHERES

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A set A in E^3 is said to have the *spherical two-piece property* (STPP) if no plane or sphere in E^3 separates A into more than two pieces. Examples of such sets include a sphere, a plane, a circle, an annulus, and a torus of revolution. In the first part of this paper we give a characterization of all closed sets in the plane with the STPP.

A surface M^2 in E^n is said to be *tight* if no hyperplane separates M^2 into more than two pieces. (This is equivalent to the hypothesis that M has minimal total absolute curvature in the sense of Chern and Lashof.) We proceed to identify all tight smooth surfaces which are subsets of spheres in E^n. By a result of Kuiper, the only tight surface in a sphere S^{n-1} which is not contained in a 3-sphere S^3 is a Veronese surface, which is an algebraic surface in S^4. We show as the main result of this paper that the only tight surfaces in S^3 other than S^2 are the images under inverse stereographic projection of cyclides of Dupin, a class of algebraic tori in E^3.

Several of the results in the first part of this paper have been generalized by S. Glass [8].

1. The circle-two-piece property

A set A in the plane E^2 has the *circle-two-piece-property* (CTPP) if every circle or straight line S in E^2 separates A into at most two pieces. In an earlier study [2] we examined objects with the ordinary two-piece property, i.e., those which are separated into at most two pieces by every line in E^2, and we related this property to the study of total absolute curvature. The CTPP is also related to total absolute curvature, but for objects in a sphere rather than in Euclidean space, and we make this relationship more precise in the latter sections of this paper. In this section however, we give a treatment which is independent of the total absolute curvature theory, and also independent of the ordinary TPP.

The whole space E^2 has the CTPP, since any circle or straight line S determines precisely two open complementary components D_1 and D_2, with closures $\bar{D}_i = D_i \cup S, i = 1, 2$. The empty set, a one-point set, and a set consisting of just two points all have the CTPP trivially. The last-mentioned

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