J. DIFFERENTIAL GEOMETRY
4 (1970) 169-185

TEICHMÜLLER THEORY FOR SURFACES WITH BOUNDARY

C. J. EARLE & A. SCHATZ

1. Introduction

(A) Recently Earle and Eells [9] determined the homotopy types of the diffeomorphism groups of closed surfaces. Here similar methods are applied to compact surfaces with boundary. As in [9] we form a principal fibre bundle whose total space consists of the smooth conformal structures on the surface, whose base is the reduced Teichmüller space, and whose structure group is a group of diffeomorphisms of the surface. Again, as in [9], we rely on a new theorem about continuous dependence on parameters for solutions of Beltrami equations. The proof of that theorem is given in § 8. The remainder of the paper can be read independently of § 8, but the reader will find it helpful to consult [9]. Fuller accounts of Teichmüller theory may be found in [2], [5], [10], [13].

(B) Now we shall state our main theorems. Let X be a smooth (C^∞) surface with boundary, and denote by $\mathscr{D}(X)$ the topological group of all diffeomorphisms of X, with the C^∞-topology of uniform convergence on compact sets of all differentials. $\mathscr{D}_0(X)$ is the subgroup consisting of the diffeomorphisms which are homotopic to the identity and map each boundary curve onto itself, preserving orientation. We shall find later that $\mathscr{D}_0(X)$ is the arc component of the identity in $\mathscr{D}(X)$.

We denote by $\mathscr{M}(X)$ the space of smooth conformal structures on X, again with the C^∞ topology. There is a natural action

$$\mathscr{M}(X) \times \mathscr{D}(X) \to \mathscr{M}(X)$$

defined by letting $\mu \cdot f$ be the pullback of the metric μ by the diffeomorphism f.

Theorem. *Assume that X is compact and orientable and that the Euler characteristic $e(X)$ is negative. Then*

(a) *$\mathscr{M}(X)$ is a contractible Fréchet manifold,*

(b) *$\mathscr{D}_0(X)$ acts freely, continuously, and properly on $\mathscr{M}(X)$,*

(c) *the quotient map*

Communicated by James Eells, Jr., May 2, 1969, and in revised form, July 30, 1969. This research was partially supported by the National Science Foundation grants GP-8219 and GP-8413.