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WEYL MANIFOLDS

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In 1918 H. Weyl [6] introduced a generalization of Riemannian geometry in his attempt to formulate a unified field theory. Weyl's theory failed for physical reasons, but it remains a beautiful piece of mathematics, and it provides an instructive example of non-Riemannian connections. In § 1 of this paper we summarize the classical definitions and theorems concerning Weyl structures; in § 2 we show that a Weyl structure is equivalent to a connection on a certain line bundle, prove the classical results using modern machinery and notation (following Kobayashi & Nomizu [4] and Nelson [5]), and derive a characterization of Weyl structures in terms of their induced linear connections.

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1. Summary of classical results

The physical motivation for Weyl's ideas is as follows. In the general theory of relativity, Einstein used Riemannian geometry as a model for physical space. However, the universe is not really a Riemannian manifold, for there is no absolute measure of length; that is, instead of being given a scalar product on the tangent space at each point, we are given a scalar product determined only up to a positive factor at each point. This fact produces no essential change in the geometry provided that a determination of length at one point uniquely induces a determination of length on the whole manifold, i.e., if it makes sense to compare the size of two tangent vectors at two distinct points. Weyl conjectured that this is not the case; rather, that an analogy should be drawn with the theory of linear connections, in which it generally makes sense to say that two vectors at two distinct points have the same direction only if there is specified a curve between the two points along which "parallel translation" can take place. Hence in the Weyl theory a determination of length at one point induces only a first-order approximation to a determination of length at surrounding points. We proceed to make these ideas precise.

M will always denote an n-dimensional smooth manifold, T_pM the tangent

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