J. DIFFERENTIAL GEOMETRY
4 (1970) 133-144

RIGIDITY AND CONVEXITY OF HYPERSURFACES IN SPHERES

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1. Introduction

We shall consider isometric immersions $x\colon M^n \to X^{n+1}$ of a compact, connected, orientable, n-dimensional ($n \geq 2$), C^∞ Riemannian manifold M^n in a simply connected Riemannian manifold X^{n+1} of constant sectional curvature. When X^{n+1} is the Euclidean space E^{n+1}, the following results, usually associated with the names of Hadamard and Cohn-Vossen, are known.

If M^n has non-negative sectional curvatures, then

(a) $x(M^n) \subset E^{n+1}$ is the boundary of a convex body; in particular, x is an embedding and M^n is diffeomorphic to the unit sphere $S^n \subset E^{n+1}$.

(b) If $y\colon M^n \to E^n$ is another isometric immersion, then x and y differ by a rigid motion (isometry) of E^{n+1}.

Part (a) of the above result, for $n=2$ and positive curvature, was first proved by Hadamard [8]. The case $n = 2$ with non-negative curvature was considered by Chern-Lashof [6], and the result for arbitrary n follows from papers by van Heijenoort [16] and Sacksteder [14]; a simple direct proof can be found in do Carmo-Lima [7].

Part (b) for $n=2$ and positive curvature is the famous Cohn-Vossen theorem; a simple proof due to Herglotz may be found in [5]. The case $n = 2$ with non-negative curvature was considered by K. Voss [17] and, independently, by Pogorelov [11]. A proof for the general case follows from Sacksteder [15, Theorem V].

In this paper we shall primarily consider the case where X^{n+1} is the unit sphere $S^{n+1} \subset E^{n+2}$ with its canonical metric of constant sectional curvature equal to one. (We indicate briefly in § 5 the situation obtained when X^{n+1} is a simply connected manifold of constant negative curvature.) A natural substitute for the above condition on the sectional curvatures is the requirement that the sectional curvatures of M^n be greater than or equal to one (the curvature of the ambient space). We shall prove that the Hadamard and Cohn-Vossen results have exact analogues in this case. Precisely, we prove the following

1.1. Theorem. *Let $x\colon M^n \to S^{n+1}$ be an isometric immersion of a compact, connected, orientable n-dimensional C^∞ Riemannian manifold into the*

Communicated by S. S. Chern, April 17, 1969. The first author was a Guggenheim fellow supported partially by NSF grant GP 6974 and C.N.Pq., and the second author was supported in part by NSF grants GP 6895 and GP 6974.