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SUBMANIFOLDS WITH A REGULAR PRINCIPAL NORMAL VECTOR FIELD IN A SPHERE

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Introduction

In [10], the author defined a principal normal vector for a submanifold M in a Riemannian manifold $\bar{M}$. This concept is a generalization of the principal normal vector for a curve and the principal curvature for a hypersurface. In fact, if M is a hypersurface, let $\Phi(X, Y)$ be the value of the 2nd fundamental form for any tangent vector fields X and Y of M. Then, we have

$$\begin{aligned}\Phi(X, Y)e &= -\langle \bar{\nabla}_X e, Y\rangle e \\ &= \text{normal part of } \bar{\nabla}_X Y \equiv T_X Y\ ,\end{aligned}$$

where e is the normal unit vector field and $\bar{\nabla}$ is the covariant differentiation of $\bar{M}$. If λ is a principal curvature at a point x of M and X is a principal tangent vector at x corresponding to λ, then we have

$$T_X Y = \langle X, Y\rangle \lambda e \qquad \text{at } x\ .$$

If we consider λe as the principal normal vector at x of M, then the above concepts for curves and hypersurfaces are in the same category.

In [10], the author investigated the properties of the integral submanifolds in M for the distribution corresponding to a regular princial normal vector field of M in an $\bar{M}$ of constant curvature. In the present paper, the properties of M will be investigated for admitting a regular principal normal vector field, and then the results will be applied to the case in which $\bar{M}$ is a sphere and M is minimal and has two principal normal vector fields such that the corresponding principal tangent spaces span the tangent space of M. Theorem 4 in this paper is a generalization of Theorems 3 and 4 in [9].

1. Preliminaries

We will use the notation in [10]. Let $\bar{M} = \bar{M}^{n+p}$ be an $(n+p)$-dimensional C^∞ Riemannian manifold of constant curvature $\bar{c}$, and $M = M^n$ an n-dimensional C^∞ submanifold immersed in $\bar{M}$ by an immersion $\psi\colon M \to \bar{M}$ which has

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