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THE CLASSIFICATION OF REAL PRIMITIVE INFINITE LIE ALGEBRAS

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The objects under consideration are the abstract transitive infinite Lie algebras (TILA) denoted (L, L^0) which are introduced in [4]. If we let $V = L/L^0$, then the realization theorem of [4] proves that (L, L^0) is topologically isomorphic to a subalgebra of $D(V)$, the continuous derivations of the formal power series $F(V^*)$. The complex primitive TILA have been classified.

We begin by recalling this classification. The results of [3] show that $g_L^0 = L^0/L^1$ contains elements of rank one. This fact is used to show that g_L^0 is one of the following: 1. $\mathrm{sl}(n, C)$, 2. $\mathrm{gl}(n, C)$, 3. $\mathrm{sp}(n, C)$, 4. $\mathrm{Csp}(n, C)$ $(\mathrm{sp}(n, C) + \{I\})$, or that there is a formal 1-form of maximal rank such that the principle module generated by it over $F(V^*)$ is preserved under Lie derivation by elements of L.

In [6] these possibilities are analyzed and the following results established:

1. If the linear isotropy algebra $(= g_L^0)$ is $\mathrm{sl}(n, C)$, then L is the algebra of all vector fields with divergence zero.

2. If the linear isotroply algebra is $\mathrm{gl}(n, C)$, then L is either the algebra of vector fields of constant divergence or the algebra of all vector fields.

3. If the linear isotropy algebra is $\mathrm{sp}(n, C)$, then L is the algebra of all Hamiltonian vector fields.

4. In the last case L is the contact algebra.

We plan to classify the real primitive TILA using these results. We begin with a theorem of Guillemin proved in [1].

Theorem 1. *Any primitive* TILA (L, L^0) *contains a closed ideal* I *of finite codimension such that* $(I, I \cap L^0)$ *is a primitive, simple* TILA.

If (L, L^0) is a TILA, we define a filtered derivation to be a derivation d for which there exists an integer $-1 \leq i < \infty$ such that $d: L^k \to L^{k+i}$ for all k.

Corollary. *Any primitive* TILA *is contained in the algebra of filtered derivations of a primitive simple* TILA, *and contains the algebra of inner derivations as a closed ideal of finite codimension.*

Note that the filtered derivations of a TILA (M, M^0) form a filtered algebra Δ. Set $g_\Delta^i = \Delta^i/\Delta^{i+1}$. $g_\Delta^0 \circledS g_\Delta^{-1}$ is mapped injectively into the derivations of $g_M^0 \circledS g_M^{-1}$ which preserve g_M^{-1} which we shall denote by $\mathrm{Der}_*(g_M^0 \circledS g_M^{-1})$.

The program for completing the classification of real primitive TILA is to determine

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