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CONTACT RIEMANNIAN SUBMANIFOLDS

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Introduction

In a previous paper [3] the author studied a submanifold of codimension 2, which inherits a contact Riemannian structure from the enveloping contact Riemannian manifold.

In the present paper, the author generalizes the results obtained in [3] to submanifolds of codimension greater than 2. In § 1 we recall first of all the definition of contact Riemannian manifolds and some identities which hold in such manifolds, and in § 2 we give some formulas which hold for submanifolds in a Riemannian manifold. After these preliminaries, § 3 contains some identities which hold for submanifolds in a contact Riemannian manifold. In § 4 we define the notion of contact Riemannian submanifolds in the same way as given in [3]. In § 5 we define an F-invariant submanifold and study the relations between contact Riemannian submanifolds and F-invariant submanifolds.

§ 6 is devoted to a condition for a submanifold to be a contact Riemannian manifold. In the last section, § 7, we introduce the notion of normal contact submanifolds in a normal contact manifold, and obtain a condition for a contact Riemannian manifold to be a normal contact manifold.

1. Contact Riemannian manifolds

A $(2n+1)$-dimensional differentiable manifold $\tilde{M}$ is said to have a *contact structure* and called a *contact manifold* if there exists a 1-form $\tilde{\eta}$, to be called the *contact form*, on $\tilde{M}$ such that

$$\tilde{\eta} \wedge (d\tilde{\eta})^n \neq 0 \tag{1.1}$$

everywhere on $\tilde{M}$, where $d\tilde{\eta}$ is the exterior derivative of $\tilde{\eta}$, and the symbol $\wedge$ denotes the exterior multiplication.

In terms of local coordinate $\{y^\kappa\}$ of $\tilde{M}$ the contact form $\tilde{\eta}$ is expressed as

$$\tilde{\eta} = \eta_\lambda dy^\lambda \,. \tag{1.2}$$

Since, according to (1.1), the 2-form $d\tilde{\eta}$ is of rank $2n$ everywhere on $\tilde{M}$, we can find a unique vector field ξ^κ on $\tilde{M}$ satisfying

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