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SOME PROPERTIES OF NEGATIVE PINCHED RIEMANNIAN MANIFOLDS OF DIMENSIONS 5 AND 7

GRIGORIOS TSAGAS

1. Let M be a compact orientable Riemannian manifold, and denote by $K^p(M, \boldsymbol{R})$ the vector space of Killing p-forms of the manifold M over the field $\boldsymbol{R}$ of real numbers. It has been shown [3] that if the manifold M is negative k-pinched and of even dimension $n = 2m$ (resp. odd dimension $n = 2m + 1$), and $k > 1/4$ (resp. $k > 2(m-1)/(8m-5)$), then $K^2(M, \boldsymbol{R}) = 0$. In this paper, we have improved the above result for negative pinched manifolds of dimensions 5 and 7.

2. We consider a compact orientable negative k-pinched Riemannian manifold M. If α, β are two exterior p-forms of the manifold, then the local product of the two forms α, β and the norm of α are defined by

$$(\alpha, \beta) = \frac{1}{p!}\alpha^{i_1\cdots i_p}\beta_{i_1\cdots i_p} = \frac{1}{p!}\alpha_{i_1\cdots i_p}\beta^{i_1\cdots i_p} ,$$

$$|\alpha|^2 = \frac{1}{p!}\alpha^{i_1\cdots i_p}\alpha_{i_1\cdots i_p} .$$

If η is the volume element of the manifold M, then the global product of the two exterior p-forms α, β and the global norm of α are defined by

$$\langle\alpha, \beta\rangle = \int_M (\alpha, \beta)\eta ,$$

$$\|\alpha\|^2 = \int_M |\alpha|^2\eta .$$

It is well known that the following relation holds [1, p. 187]:

$$(2.1) \qquad \langle\alpha, \Delta\alpha\rangle = \|\delta\alpha\|^2 + \|d\alpha\|^2 .$$

We also have the formula [2, p. 3]:

$$(2.2) \qquad \frac{1}{2}\Delta(|\alpha|^2) = (\alpha, \Delta\alpha) - |\nabla\alpha|^2 + \frac{1}{2(p-1)!}Q_p(\alpha) ,$$

where

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