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SOME PROPERTY OF CLOSED HYPERSURFACES IN RIEMANNIAN MANIFOLDS

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1. The main result of the present paper is the

Theorem. *Let M^3 be a three-dimensional symmetric Riemannian manifold whose sectional curvature $K(P,\sigma)$ satisfies $(1-\delta)T \leq K(P,\sigma) \leq T$, where T is a positive constant and $0 \leq \delta < 1/2$. Let M be a closed surface in M^3 with the mean curvature H satisfying $H = C$, C being a positive constant, and assume that M is strictly convex, and the second fundamental form of M is positive. Let the total volume or the total area of M be denoted by V_M, and the volume of the sebset M_L of M, where the difference of the principal curvatures exceeds $2L$, be denoted by $V(L)$. If $(1-\delta)L^2 > \delta C^2$, then $V(L)$ satisfies*

$$(0) \qquad \frac{V(L)}{V_M} \leq \frac{\delta^2 C^2}{\delta(25\delta - 16)C^2 + 8(1-\delta)(2-3\delta)L^2} .$$

Corollary. *If M^3 is a space of constant curvature with positive scalar curvature, then the surface M of the above theorem is totally umbilical.*

2. Let M^{n+1} be a Riemannian manifold of dimension $n+1$, K_{kjih} the curvature tensor of M^{n+1}, and M^n a hypersurface of M^{n+1}, whose equation is given by $x^h = x^h(u^a)$ locally. Throughout this paper all the indices run as follows: $h, i, j, k = 1, \cdots, n+1$; $a, b, c, d = \dot{1}, \cdots, \dot{n}$.

We define $B_a{}^h$ as usual by $B_a{}^h = \partial_a x^h$ where $\partial_a = \partial/\partial u^a$. From $B_a{}^h$ and the unit normal vector N^h we can construct a matrix $(B_a{}^h, N^h)$ and denote its reciprocal matrix by $(B^a{}_h, N_h)$. $g_{ba} = B_b{}^i B_a{}^h g_{ih}$ is the first fundamental tensor of M^n. Using the Van der Waerden-Bortolotti operator ∇ we get $\nabla_b B_a{}^h = h_{ba}N^h$, $\nabla_b N^h = -h_b{}^a B_a{}^h$, where h_{ba} is the second fundamental tensor of M^n. The equation of Codazzi is

$$(1) \qquad \nabla_c h_{ba} - \nabla_b h_{ca} = K_{kjih} B^{kji}_{cba} N^h ,$$

and the equation of Gauss is

$$(2) \qquad 'K_{dcba} = K_{kjih} B^{kjih}_{dcba} + h_{cb}h_{da} - h_{db}h_{ca} ,$$

where $'K_{dcba}$ is the curvature tensor of the Riemannian manifold M^n.

If M^n is a closed hypersurface, then

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