J. DIFFERENTIAL GEOMETRY
3 (1969) 457-459

EINSTEIN SPACES OF POSITIVE SCALAR CURVATURE

YOSIO MUTŌ

1. Let M be an n-dimensional compact orientable Einstein space with positive scalar curvature K. Then the concircular curvature tensor Z_{kjih} defined by

$$Z_{kjih} = K_{kjih} - \frac{K}{n(n-1)}(g_{ji}g_{kh} - g_{ki}g_{jh}) \tag{1}$$

satisfies

$$Z_{kjih}g^{ji} = 0 , \tag{2}$$

because of

$$K_{ji} = \frac{K}{n}g_{ji} .$$

The purpose of the present paper is to prove the

Theorem. *If the concircular curvature tensor satisfies the inequality*

$$\frac{1}{K}|Z_{kjih}A^kB^jC^iD^h| < \frac{2}{5n^7} \tag{3}$$

at every point of M for any set of unit vectors A, B, C, D, then the Einstein space M is a space of constant curvature.

Roughly speaking, this theorem tells that, if M_0 is a space of positive constant curvature, there exist no Einstein spaces other than M_0 in a sufficiently small neighborhood of M_0. The inequality (3) is a quite rough and modest estimation. We can get a better estimation by a more elaborate calculation.

2. In an Einstein space we have $\nabla_k K_{ji} = 0, \nabla_k K = 0$, and therefore $\nabla_l Z_{kjih} = \nabla_l K_{kjih}$. Thus by using Green's theorem and the second identity of Bianchi, we get

$$\begin{aligned} -\int_M (\nabla^l Z^{kjih})(\nabla_l Z_{kjih})dV &= \int_M Z^{kjih}\nabla^l\nabla_l K_{kjih}dV \\ &= 2\int_M Z^{kjih}\nabla^l\nabla_k K_{ljih}dV , \end{aligned}$$

Communicated by K. Yano, August 5, 1968.