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A FORMULA OF SIMONS' TYPE AND HYPERSURFACES WITH CONSTANT MEAN CURVATURE

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In a recent work [8] J. Simons has established a formula for the Laplacian of the second fundamental form of a submanifold in a Riemannian manifold and has obtained an important application in the case of a minimal hypersurface in the sphere, for which the formula takes a rather simple form. The application is made by means of the Laplacian of the function f on the hypersurface, which is defined to be the square of the length of the second fundamental form.

In the present paper, by a more direct route than Simons' we first obtain the same type of formula (see (16)) in the case of a hypersurface M immersed with constant mean curvature in a space $\tilde{M}$ of constant sectional curvature, and then derive a new formula (see (18)) for the function f which involves the sectional curvature of M. Based on this new formula our main results are the determination of hypersurfaces M of non-negative sectional curvature immersed in the Euclidean space R^{n+1} or the sphere S^{n+1} with constant mean curvature under the additional assumption that the function f is constant. This additional assumption is automatically satisfied if M is compact. We state the general results in a global form assuming completeness of M, but they are essentially of local nature.

1. Formula of Simons' type

Let $\tilde{M}$ be an $(n+1)$-dimensional space form, i.e., a Riemannian manifold of constant sectional curvature, say, c. Let $\phi\colon M \to \tilde{M}$ be an isometric immersion of an n-dimensional Riemannian manifold M into $\tilde{M}$. For simplicity, we say that M is a hypersurface immersed in $\tilde{M}$ and, for all local formulas and computations, we may consider ϕ as an imbedding and thus identify $x \in M$ with $\phi(x) \in M$. The tangent space $T_x(M)$ is identified with a subspace of the tangent space $T_x(\tilde{M})$, and the normal space $T_x^{\perp}$ is the subspace of $T_x(\tilde{M})$ consisting of all $X \in T_x(\tilde{M})$ which are orthogonal to $T_x(M)$ with respect to the Riemannian metric g. For the basic notations and formulas concerning differential geometry of submanifolds, we follow Chapter VII of Kobayashi-Nomizu [4].

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