J. DIFFERENTIAL GEOMETRY
3 (1969) 351-360

A THEOREM ON MINIMAL SURFACES

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1. Introduction

It is well known that $x: M^k \to R^{k+p}$ is a minimal immersion into the Euclidean space R^{k+p} if and only if

$$\Delta\langle a, x\rangle = 0 \qquad \text{for all vectors } a \in R^{k+p} ,$$

where Δ denotes the Laplacian on M^k. This result follows immediately from the relation

$$(1) \qquad \Delta\langle a, x\rangle = -\langle a, H\rangle ,$$

where H is the mean curvature vector of the immersion. Osserman [3, Theorem 2] proved the following theorem:

If $x: M^2 \to R^3$ is such that for some $a \in R^3$, $\Delta\langle a, x\rangle = 0$, then M^2 is a minimal surface, or else a locally cylindrical surface with its generators parallel to a.

The purpose of this paper is to generalize Osserman's theorem to the case $x: M^2 \to R^n$. We will assume as a natural generalization of Osserman's hypothesis the existence of an $n-2$ dimensional subspace $A \subset R^n$ so that $\Delta\langle a, x\rangle = 0$ for all $a \in A$. The main theorem is the following:

Theorem. *Let M^2 be a connected manifold, and A an $n-2$ dimensional subspace of Euclidean space R^n. If $x: M^2 \to R^n$ is an immersion such that*

$$(2) \qquad \Delta\langle a, x\rangle = 0 \qquad \text{for all } a \in A ,$$

then there are only two possibilities:

1. *M^2 is minimally immersed in R^n, or*
2. *there is a set $\{m_i\}$ of isolated points in M so that each point of the complement M_0 of this set has a neighborhood U which is mapped onto a regular curve γ in B, the orthogonal complement of A, by the orthogonal projection π with kernel A in such a way that $x|U$ is a minimal immersion in the cylinder $\gamma \times A$.*

Conversely, if the immersion $x: M^2 \to R^n$ is as described under 1 *or* 2, *it satisfies the hypothesis of the theorem.*

The type of immersion described under point 2 will for convenience be referred to as type 2. Similarly for type 1.

Received August 15, 1968. This research was supported by the National Research Council of Canada.