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COMPLEX HYPERSURFACES OF A COMPLEX PROJECTIVE SPACE

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1. Statement of results

Let M be a compact complex hypersurface of the complex projective space $P_{n+1}(C)$. Then by a well known theorem of Chow, M is algebraic. We shall prove the following theorems.

Theorem 1. *Let M be a compact complex hypersurface of the complex projective space $P_{n+1}(C)$, and suppose that the Euler-Poincaré characteristic $\chi(M)$ of M is $n + 1$. Then*

(1) *M is a complex hyperplane $P_n(C)$ if n is even.*

(2) *M is either a complex hyperplane $P_n(C)$ or a complex hyperquadric in $P_{n+1}(C)$ if n is odd.*

Theorem 2. *Let M be a complete complex hypersurface of the complex projective space $P_{n+1}(C)$. If every holomorphic sectional curvature of M is greater than $1/2$ with respect to the metric induced from the Fubini-Study metric of $P_{n+1}(C)$, then M is a complex hyperplane $P_n(C)$.*

It should be remarked that the referee of this paper has made the following conjecture stronger than Theorem 2: Let M be a complete complex hypersurface of the complex projective space $P_{n+1}(C)$. If M admits a Kaehler metric with respect to which M is of holomorphic pinching greater than $1/2$, then M is a complex hyperplane $P_n(C)$.

Theorem 3. *Let M be a compact complex hypersurface of the complex projective space $P_{n+1}(C)$. If every holomorphic sectional curvature of M is positive with respect to the metric induced from the Fubini-Study metric of $P_{n+1}(C)$, then M is either a complex hyperplane $P_n(C)$ or a complex hyperquadric in $P_{n+1}(C)$.*

2. Proof of Theorem 1

Let h be the generator of $H^2(P_{n+1}(C), Z)$ corresponding to the divisor class of a hyperplane $P_n(C)$. Then the total Chern class $c(P_{n+1}(C))$ of $P_{n+1}(C)$ is given by

$$c(P_{n+1}(C)) = (1 + h)^{n+2} .$$

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