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AN INTEGRAL FORMULA FOR IMMERSIONS IN EUCLIDEAN SPACE

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1. Introduction

This paper derives a general rigidity theorem and an integral formula for immersions of a compact oriented riemannian manifold without boundary in a euclidean space. The formula is applied to a volume-preserving immersion to establish a simple geometric criterion that the immersion be isometric. As the integral formula has a formal resemblance to one derived by Chern and Hsiung in [1], we conclude the paper with some remarks about that work.

2. Notations and conventions

Let M be a compact oriented m-dimensional riemannian manifold without boundary with metric $ds^{2\#}$, and let

$$X: M \to R^{m+n}$$

be an immersion in an $(m+n)$-dimensional euclidean space R^{m+n}. As such M admits a second riemannian metric,

$$ds^2 = dX \cdot dX \ .$$

We fix the range of indices so that the capital Latin indices run from 1 to $m+n$, the small Greek indices from 1 to m, and the small Latin indices from $m+1$ to $m+n$.

Matters being so, we choose orthonormal coframes $\{\tau^{\alpha\#}\}$ for $ds^{2\#}$ on M which diagonalize ds^2 with respect to $ds^{2\#}$. Thus

$$ds^{2\#} = \Sigma(\tau^{\alpha\#})^2 \ , \qquad ds^2 = \Sigma g_\alpha(\tau^{\alpha\#})^2 \ ,$$

and the first invariants of the pair of metrics are the elementary symmetric functions in the functions g_α.

Next we choose a family of orthonormal frames $\{e_A\}$ on $X(M)$ in R^{m+n} in such a way that $\{e_\alpha\}$ are unit tangent vectors of $X(M)$ and the pull back of the dual coframe $\{\tau^A\}$ satisfies

$$\tau^\alpha = h_\alpha \tau^{\alpha\#} \ ,$$

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