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TRANSFORMATION GROUPS ON RIEMANNIAN SYMMETRIC SPACES

TAKUSHIRO OCHIAI

1. On any Riemannian space N, we denote by $I(N)$ (resp, by $I(N)^0$) the group of all isometries (resp. the identity connected component of $I(N)$). The purpose of this note is to prove the following results.

Theorem 1. *Let M be a Riemannian symmetric space of the noncompact type, and L a (not necessary connected) effective Lie transformation group on M. If* $L \supset I(M)^0$, *then* $I(M) \supset L$.

Corollary (*E. Cartan*). *Let M be a Riemannian symmetric space of the noncompact type. Then* $I(M)$ *is isomorphic to the group of all automorphisms of* $I(M)^0$.

Theorem 2. *Let M be an irreducible Riemannian symmetric space which is not of the Euclidian type, and let* $\mathscr{D}$ *be an* $I(M)^0$*-invariant differential operator on M. Then any transformation f of M leaving* $\mathscr{D}$ *invariant is an isometry.*

Theorem 1 has been proved partially in [4]. The author wishes to thank Professor T. Nagano for his valuable and generous suggestions.

2. Denote by G_n the isotropy subgroup of any transitive transformation group G on a manifold N at any point $n \in N$, and by M a Riemannian symmetric space of the noncompact type unless stated otherwise. Then the following properties of M are well known:

(i) *M is homeomorphic to an open cell.*

(ii) *Each irreducible factor in the de Rham decomposition of M is again a Riemannian symmetric space of the noncompact type.*

(iii) $I(M)^0$ *is semi-simple.*

(iv) $I(M)^0$ *is also the identity connected component of the group of all isometries of any* $I(M)^0$*-invariant metric on M.*

(v) $I(M)^0_m$ $(m \in M)$ *is a maximal compact subgroup of* $I(M)^0$.

(vi) $I(M)^0_m \neq I(M)^0_n$ *if* $m \neq n$ $(m, n \in M)$.

A Riemannian symmetric space N (not of the Euclidian type) is irreducible if and only if $I(N)^0$ is simple, and in this case, the linear isotropy representation of $I(N)^0_n$ $(n \in N)$ is irreducible.

Lemma 1. *Let G be an effective Lie transformation group on M such that*

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