

NONABELIAN SPENCER COHOMOLOGY AND DEFORMATION THEORY

NGÔ VAN QUÊ

In memory of the author's father

Introduction

This work is a continuation of a preceding paper [9] modulo some change of notation. In particular, Chapter I on Lie groupoids is a development of the corresponding chapter of [9]. However, this work has its own interest in introducing the formalism of Nonabelian Spencer Cohomology.

Let V be a compact manifold, and Γ a transitive continuous pseudogroup on V (Definition 1.1, Chap. II). For every large integer k , one defines a fiber bundle of homogenous spaces, i.e., a fiber bundle whose fiber is a homogenous space G/H , and denotes this fiber bundle by $C_k(\Gamma)$ (Proposition 2.2, Chap. II). We intend to prove:

(1) There is an involutive differential system S_1 of order 1 in $C_k(\Gamma)$ such that every family of deformations of (V, Γ) (Definition 3.1, Chap. II) induces a family of sections in $C_k(\Gamma)$, which are solutions of the differential system S_1 .

(2) In the case where Γ is analytic and elliptic (Definition 1.2, Chap. II), every family of sections in $C_k(\Gamma)$, which are solutions of the differential system S_1 , defines inversely a family of deformations of (V, Γ) .

The last result is based essentially on the Malgrange-Newlander-Nirenberg theorem (Theorem 2.2, Chap. II), the proof of which not to be given here follows from an argument to be published by B. Malgrange, reproving in particular the well-known theorem of A. Newlander and L. Nirenberg on the "integrability" of almost complex structures.

However, we completely reformulate in our formalism an argument of M. Kuranishi [7] proving the existence of an analytic space K of finite dimension, which is a "locally universal space of deformations" for elliptic pseudogroups (Theorem 4.1, Chap. II). This space was known to M. Kuranishi in the case of complex analytic structures.

This work was developed during a seminar held in 1966-67 under the direction of D. C. Spencer at Stanford University. The central ideas were