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VECTOR FORMS AND INTEGRAL FORMULAS FOR HYPERSURFACES IN EUCLIDEAN SPACE

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Introduction

Let Σ be a smooth oriented m-dimensional hypersurface immersed in $(m+1)$-dimensional Euclidean space E^{m+1}. In § 2, we consider some vector form invariants for Σ and their expansions in terms of elementary symmetric functions of pricipal curvatures and certain intrinsic tangent vectors. We use these results in § 3 to obtain integral formulas for Σ assuming that Σ has closed regular boundary. For a compact Σ we have integral formulas of particular interest in Corollary 2 of Theorem 3.1; these are similar to Minkowski formulas and involve gradients of elementary symmetric functions of principal curvatures. Some consequences of these formulas are studied in § 4. In Theorem 3.3 we prove that for a compact hypersurface of constant mean curvature, the surface integral of the gradient of any elementary symmetric function of principal curvatures is identically zero.

1. Preliminaries

Let M be an oriented smooth differentiable manifold of dimension m. Our hypersurface Σ is a mapping $X: M \to E^{m+1}$ where the Jacobian matrix has rank m everywhere. Let $n(x)$, $x \in M$, be a unit normal to Σ at $X(x)$. Then choosing an orthonormal frame $e_1, \cdots, e_m$ in the tangent space of Σ at $X(x)$ such that the det $(e_1, \cdots, e_m, n) = 1$, we have

$$dX = \sum_i \sigma_i e_i \, , \qquad dn = \sum_i \omega_i e_i \, , \tag{1.1}$$

where σ_i and ω_i are differential 1-forms. We express ω_i in terms of the linearly independent σ_i:

$$\omega_i = \sum_j a_{ij}\sigma_j \, , \tag{1.2}$$

where $\|a_{ij}\|$ is symmetric.

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