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REMARKS ON THE FIRST MAIN THEOREM IN EQUIDISTRIBUTION THEORY. III

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1. On C^n, there are two canonical convex exhaustions, $\tau_0 = \Sigma_i z_i \bar{z}_i$ and $\tau_1 = \log(1 + \Sigma_i z_i \bar{z}_i)$. Up to scalar factors, $dd^c\tau_0$ is the kähler form of the flat metric on C^n, and $dd^c\tau_1$ is the kähler form of the pull-back of the Fubini-Study metric by the inclusion $C^n \subseteq P_nC$. (We also call the latter metric the *Fubini-Study metric on* C^n.) Considering the way $dd^c\tau$ enters into the condition which guarantees that the complement of the image under a holomorphic $f\colon C^n \to M$ be of measure zero (Theorem 5.1, Part II [13]), we readily appreciate the importance of these exhaustion functions. It seems that much attention has thus far been lavished on τ_0, and this is unjustified—and perhaps unjustifiable. For instance, the inclusion $C^n \subseteq P_nC$, which is of course *quasi-surjective* ($\equiv$ surjective up to a set of measure zero), does not satisfy the condition stipilated in equation (12) of Corollary 5.2 in Part II [13]. What we would like to contend here is that τ_1 should be given a more prominent position than τ_0 in equidistribution theory. The critical feature of τ_1 which accounts for its usefulness is that $\int_{C^n} (dd^c\tau_1)^n$ is finite, i.e., the Fubini-Study metric induces a totally finite measure on C^n. The theorems proved in this note, all of them intuitively plausible, purport to demonstrate this fact.

There is nevertheless one place where the flat metric on C^n (and hence τ_0) might prove to be useful. This is the case of a holomorphic $f\colon C^n \to C^n$; the situation is most natural (e.g. the Faton-Bieberbach example) and it seems legitimate that one should capitalize on the simplicity of the flat metric by carrying out all investigations in terms of it. The usual procedure of first imbedding C^n into P_nC and then considering the composite map $f\colon C^n \to P_nC$ becomes in this light less desirable. For this reason, a proof, directly exploiting the flat metric, of the first main theorem with C^n as image manifold is given in the appendix. It turns out that no harmonic theory is needed for this case, but the theorem so obtained is not very strong. Time will decide whether further work should be done in this direction, or whether this undertaking should be abandoned altogether.

2. Let us begin by recalling some elementary facts about C^n and P_nC. On

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