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REDUCIBILITY OF EUCLIDEAN IMMERSIONS OF LOW CODIMENSION

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1. Introduction

By a theorem of Kobayashi, the holonomy algebra of a compact D-dimensional Riemannian manifold M, isometrically immersed in Euclidean space R^{D+1}, is the full orthogonal algebra (M is not reducible, therefore). Suppose M is a reducible compact D-dimensional manifold having an isometric immersion ϕ in R^{D+2}. A theorem of R. L. Bishop gives the holonomy algebra of M at m to be the sum $o(K) + o(D-K)$ of two orthogonal algebras acting on complementary orthogonal subspaces of the tangent space M_m. We show (Theorem 8.2) that, at least when ϕ is one-one, ϕ is in fact the product of two immersions of hypersurfaces, with an exception occurring in the case $K = 1$ or $D - 1$.

In § 9, certain Euclidean immersions are shown to be cylindrical. The following result, for example, follows from the codimension one case and a well-known theorem of Hartman and Nirenberg: If a complete D-dimensional manifold M has an isometric immersion ϕ in R^{D+1}, then M is a Riemannian product $M_1 \times R^{D-K}$, where the restricted holonomy group of M_1 acts irreducibly, and ϕ is $(D-K)$-cylindrical.

Throughout, M indicates a connected Riemannian manifold, and all structures are C^∞.

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2. Isometric immersions

Some basic material concerning an isometric immersion $\phi: M \to \overline{M}$ is outlined here, largely to establish notation.

Let K and $\overline{K}$ be the Riemannian tangent bundles of M and $\overline{M}$ respectively. K is identified through the tangent map $d\phi$ with a metric sub-bundle of $\overline{K} \mid M$; $K^\perp$ will be the sub-bundle with complementary orthogonal fiber over each m (we write: $M_m + M_m{}^\perp = \overline{M}_m$). Letting $\mathfrak{F}$ be the algebra of smooth

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