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ON A PROBLEM OF NOMIZU-SMYTH ON A NORMAL CONTACT RIEMANNIAN MANIFOLD

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The study of complex Einstein hypersurfaces of Kählerian manifolds of constant holomorphic sectional curvature has been initiated by Smyth [12] and continued by Nomizu and Smyth [7]. (See also, Ako [1], Chern [2], Kobayashi [5], Smyth [13], Takahashi [14], Yano and Ishihara [17]).

The main purpose of the present paper is to study the so-called invariant C-Einstein submanifolds of codimension 2 in a normal contact Riemannian manifold. We call a problem of this kind a problem of Nomizu-Smyth.

First of all we recall in §1 the definition and properties of contact Riemannian manifolds, and in §2 the fundamental formulas for submanifolds of codimension 2 in a Riemannian manifold.

In §§3, 4 we obtain the fundamental formulas respectively for submanifolds and invariant submanifolds of codimension 2 in a contact Riemannian manifold.

In the last §5, we study the problem of Nomizu-Smyth, that is, the problem of determining invariant C-Einstein submanifolds of codimension 2 in a normal contact Riemannian manifold of constant curvature.

1. Contact Riemmannian manifolds

First of all for later use we recall the definition and some properties of a contact Riemannian manifold. A $(2n+1)$-dimensional differentiable manifold M is said to admit a *contact structure* if there exists on M a 1-form $E = E_i dx^i$ such that the rank of the tensor field

$$F_{ji} = \frac{1}{2}(\partial_j E_i - \partial_i E_j) \tag{1.1}$$

is $2n$ everywhere on M, where ∂_i denotes the operator $\partial/\partial x^i$, (x^h) are the local coordinates of M, the indices $h, i, j, k, \cdots$ run over the range $\{1, \cdots \cdots, 2n+1\}$, and the so-called Einstein's summation convention is used with respect to this system of indices. A manifold admitting a contact structure is called a *contact manifold*.

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