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AFFINE AND RIEMANNIAN s-MANIFOLDS

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1. Introduction

Let M be a connected Riemmannian manifold, and $I(M)$ the group of all isometries on M. An isometry on M with an isolated fixed point x will be called a *symmetry* at x, and will usually be written as s_x. A point x is an isolated fixed point of a symmetry s_x if and only if s_x induces on the tangent space M_x at x an orthogonal transformation $S_x = (ds_x)_x$ which has no invariant vector. M will be called an *s-manifold* if for each $x \in M$ there is a symmetry s_x at x.

An important case arises when each s_x has order 2. Then M is a symmetric space and $I(M)$ is transitive. Indeed, s_x is the geodesic symmetry at x and the set of all such geodesic symmetries is transitive. It will be shown that the transitivity of $I(M)$ is an implication of the existence of a symmetry s_x at each point x without the assumption of s_x being involutive. Thus we have

Theorem 1 (*F. Brickell*). *If M is a Riemannian s-manifold, then I(M) is transitive.*

The assignment of a symmetry s_x at each point x can be viewed as a mapping $s: M \rightarrow I(M)$, and $I(M)$ can be topologised so that it is a Lie transformation group [1]. In this theorem, however, no further assumption on s is made; even continuity is not assumed.

A symmetry s_x will be called a *symmetry of order* k at x if there exists a positive integer k such that $s_x^k = Id.$, and a Riemannian s-manifold with a symmetry of order k at each point will be called a *Riemannian s-manifold* of order[1] k. Clearly a Riemannian s-manifold of order 2 is a symmetric space in the ordinary sense.

Let M be a connected manifold with an affine connection, and $A(M)$ the Lie transformation group of all affine transformations of M. An affine transformation s_x will be called an *affine symmetry* at a point x if x is an isolated fixed point of s_x. The proof of Theorem 1 does not extend to a manifold with affine symmetries. However, assuming differentiability of the mapping $s: M \rightarrow A(M)$, we obtain a similar result. A connected manifold with an affine con-

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[1] The concepts of a Riemannian s-manifold and a Riemannian s-manifold of order k were introduced in [2] for the case when the map $s: M \rightarrow I(M)$ is differentiable.