J. DIFFERENTIAL GEOMETRY
2 (1968) 391-392

FOUR BRIEF EXAMPLES CONCERNING POLYNOMIALS ON CERTAIN BANACH SPACES

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Let E denote one of the spaces l^p $(1 \leq p < \infty)$ or c_0, and $\{e_1, e_2, \cdots\}$ be the standard basis in E. An element x in E will be written as $x = \Sigma_n x_n e_n$. The following examples are perhaps justified by the fact that their proofs are shorter than their statements.

Example A. *Suppose E is real*, and

$$\phi(t) = 3t^2 - 2t^3 \ (t \text{ real}), \qquad \phi_n(t) = \phi(\alpha_n t)/2^{n-1},$$

where $\alpha_n = 2^{n/4}$. *Then the mapping* $A(x) = \Sigma_n \phi_n(x_n)$ *is a continuous real-valued polynomial of degree* 3, *and the image of the critical points contains* $[0, 2]$.

Proof. Any x of the form $x = \Sigma_n \varepsilon_n \alpha_n^{-1} e_n$, where ε_n is 0 or 1, is a critical point of A, and $A(x) = \Sigma_n \varepsilon_n / 2^{n-1}$.

Example A is based on examples of Kupka [2] and Bonic [1], and the remark "bien sur" of Douady [Baton Rouge, April 1967].

Example B. *Suppose E is complex, and*

$$\begin{aligned} &\psi(z) = az^2 + bz^3 + cz^4 + dz^5, \\ &\psi_n(z) = \psi(\beta_n z)/2^{n-1}, \\ &\quad 4a = 4i + 4, \qquad 4b = -5i + 5, \\ &\quad 4c = -2i - 2, \qquad 4d = 3i - 3, \qquad \beta_n = 2^{n/6}, \end{aligned}$$

where z is complex. Then the mapping

$$B(x) = \Sigma_n \psi_n(x_{2n-1}) + \Sigma_n \psi_n(x_{2n})$$

is a continuous complex-valued polynomial of degree 5, *and the image of the critical points contains* $[0, 2] \times [0, 2]$.

Proof. Any x of the form

$$x = \Sigma_n \varepsilon_n \beta_n^{-1} e_{2n-1} - \Sigma_n \delta_n \beta_n^{-1} e_{2n},$$

Communicated by R. S. Palais, March 18, 1968. Research supported in part by NSF Grant GP-7026.