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REMARKS ON THE FIRST MAIN THEOREM IN EQUIDISTRIBUTION THEORY. II

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To Ambrose in gratitude and esteem

1. The purpose of this paper is to integrate the non-integrated First Main Theorem ((i) of the Theorem in Part I [12]) for the case of a holomorphic map $f: V \to M$, where V is an open complex manifold and M is compact kählerian and of the same dimension n as that of V. The formal aspect of the integration is trivial, but the more delicate part is to attend to the convergence of the improper integrals which thus arise. In general, the convergence question is quite untractable, as contrasted with the case of Riemann surfaces where such matters can be adequately treated without exception [8]. The principal results of this work are Theorem 5.1 and Corollary 5.2; the simplicity and the very geometric nature of the hypothesis in these results justify the approach adopted here.

I take this opportunity to point out that the proof of the Theorem in [12] is incomplete at two places; the last section (§8) of this paper is devoted to filling in these gaps.

2. Definition 2.1. Let V be an open complex manifold. *A* C^∞ function $\tau: V \to [0, \infty)$ is called an *exhaustion function* iff

(i) τ is proper, i.e. τ^{-1} (compact set) is compact,

(ii) τ has only isolated critical points in $\tau^{-1}[r_0, \infty)$ for some r_0.

A first remark is that every open manifold, real or complex, always admits an exhaustion function (in fact one with only nondegenerate critical points). *We will always work in the range* $[r_0, \infty)$. Let $\mathscr{C}$ be the set of critical values of τ in $[r_0, \infty)$. Then (ii) says that $\mathscr{C}$ is discrete. If $t \notin \mathscr{C}$, then $\tau^{-1}(t)$ is a compact submanifold of V by (i). If $t \in \mathscr{C}$, then $\tau^{-1}(t)$ is a compact set which, with a finite number of points deleted, is a submanifold of V. We will consistently employ the notation:

$$V[t] = \tau^{-1}[0, t],$$
$$\partial V[t] = \tau^{-1}(t) .$$

It should be emphasized that the parameter value of the exhaustion functions

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