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POSITIVELY-CURVED HYPERSURFACES OF A HILBERT SPACE

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1. Statement of the results

A *Riemannian Hilbert manifold M* is a differentiable (C^∞), connected manifold, modeled on a (separable) Hilbert space, such that in the tangent space M_p of M at each $p \in M$ there exists an inner product $\langle , \rangle_p$, which varies differentiably with p (see [7] for precise definitions). M can be made into a metric space by defining the distance between two points $p, q \in M$ as the infimum of the length of differentiable curves joining p and q; M is said to be *complete* if it is complete in this metric.

The local differential geometry of Riemannian Hilbert manifolds develops in exactly the same way as in the finite dimensional case so that we can define a unique covariant derivative and obtain the notions of curvature tensor, geodesics, sectional curvature, etc. (see [8] for details). It can be proved, for example, that convex neighborhoods exist for each point of M [8, pp. 14–16].

However, for the global differential geometry, the situation is quite different. Only a few theorems are known, the main reason being that completeness does not imply, as it does in the finite dimensional case, that two given points of M can be joined by a minimal geodesic; a simple example is given in [3].

The objective of this paper is to prove a global result, for the statement of which we need a few definitions.

A differentiable (C^∞) map $x: M \to H$ of a Riemannian Hilbert manifold M into a Hilbert space H is an *immersion* if the differential $dx(p): M_p \to H$ is one-one and $dx(p)(M_p) \subset H$ is closed in H. If x is one-one it is called an *embedding*.

An *isometric immersion* is an immersion $x: M \to H$ such that $dx(p): M_p \to H$ is an isometry for each $p \in M$. If, in this situation, $dx(p)(M_p) \subset H$ has codimension one, we say that $x(M) \subset H$ is a *hypersurface* of H. Of course, a hypersurface may have self-intersections.

We now state the theorem, which will be proved in §4.

Theorem. *Let M be a complete Riemannian Hilbert manifold with positive sectional curvature K bounded away from zero at each point of M, i.e., for*

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