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THE HOLONOMY ALGEBRA OF IMMERSED MANIFOLDS OF CODIMENSION TWO

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1. Introduction

In [4] S. Kobayashi has proved that the holonomy algebra of a compact riemannian manifold immersed in euclidean space of one dimension greater is the whole orthogonal algebra. The purpose of this paper is to generalize this result to the case of codimention two and, to a certain extent, the noncompact case. Our technique gives a simple proof of Kobayashi's theorem. The extensions to codimension two are as follows.

Theorem 1. *Let M be a riemannian manifold of dimension D isometrically immersed in a euclidean space R^{D+2} of dimension $D+2$. Then there are the following possibilities for $r(m)$, the Lie algebra generated by the curvature transformations at a point $m \in M$:*

(a) *The relative curvature space $k(m)$ at m decomposes into an orthogonal direct sum, $k(m) = V + W$, and $r(m) = o(V) + o(W)$, the direct sum of the orthogonal algebras based on V and W. (V or W may be of dimension zero or one, so that $r(m)$ is itself an orthogonal algebra.) Or:*

(b) *There is a complex structure on $k(m)$, $r(m)$ is the unitary algebra of that structure, and the second fundamental forms are all of signature zero, unless $\dim k(m) = 4$.*

In particular, if there is a point m at which some second fundamental form is nondegenerate, that is, $k(m) = M_m$, then the global holonomy algebra h_m at m is one of the possibilities listed.

Theorem 2. *Let M be a compact riemannian manifold of dimension $D \neq 4$ isometrically immersed in R^{D+2}. Then $h_m = o(V) + o(W)$, where $M_m = V + W$ is an orthogonal direct sum.*

The results in this paper are algebraic, except for a minor point used in passing from Theorem 1 to Theorem 2. They have been used as a starting point by Stephanie B. Alexander [1] to show that under the hypothesis of Theorem 2 the immersion is usually the product of two hypersurface immersions.

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