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SOME FROBENIUS THEOREMS IN GLOBAL ANALYSIS

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Introduction

In [6] we introduced a notion of differentiability which permitted us to prove that the group of C^∞ diffeomorphisms can be given the structure of a Lie group. This notion of differentiability as distinct from the Frechet definition does not depend on a topological or quasi-topological structure on the vector space of continuous linear transformations $L(E, F)$ between topological vector spaces E, F (see § 1 below). However, in [6], to prove the fundamental elementary theorems of analysis, we used the notion of quasi-topology introduced by A. Bastiani.

In § 1 it is shown how these theorems can be established by elementary techniques.

In § 2 a version of the Frobenius theorem is proved (see Theorem 3). Although our proof of Theorem 3 differs in several essential points from an analogous proof in Dubinsky [4] of an analogous theorem, we found his ideas quite useful. In Proposition 6 it is proved that under the hypotheses of Theorem 3 a C^n differential equation admits a C^n flow.

In § 3 a second version of the Frobenius theorem is proved in the context of Banach chains.

In § 4 a Frobenius theorem on the integrability of finite codimensional sub-bundles of the tangent bundle of manifolds modelled on Banach chains is proved.

In § 5 there is given an application of §§ 3 and 4 in the context of the group of diffeomorphisms of a compact connected smooth manifold; there, it is shown that finite dimensional and finite codimensional subalgebras of the Lie algebra of the right invariant vector fields on Diff (M) are integrable.

Corollaries 1 and 2 of Theorem 5 were pointed out to us in a letter by C. J. Earle and J. Eells. The author wishes to express his appreciation to the referee for his valuable suggestions and numerous helpful comments.

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