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NON-PARAMETRIC HYPERSURFACES WITH BOUNDED CURVATURES

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1. Introduction

We work with the hypersurface in E^{n+1} which is the graph of a C'' function

$$u = u(x^1, \cdots, x^n)$$

on an open ball $\sum(x^i)^2 < R^2$ in E^n. We use the notation

$$p_i = \partial u/\partial x^i, \qquad r_{ij} = \partial^2 u/\partial x^i \partial x^j,$$
$$p = (p_1, \cdots, p_n), \qquad R = \|r_{ij}\|,$$
$$w^2 = 1 + |p|^2 = 1 + \sum p_1^2.$$

We also introduce the matrix

$$A = -\frac{1}{w} R\left(I - \frac{1}{w^2}\,{}^t pp\right).$$

It is known that A has geometrical significance, indeed, its characteristic roots are the principal curvatures of the hypersurface (see Flanders [3, pp. 116–126] for details). The various curvatures $K_1 =$ mean curvature, $K_2, \cdots, K_n (=$ total curvature) are given by the characteristic polynomial:

$$\det(tI - A) = t^n - \binom{n}{1} K_1 t^{n-1} + \binom{n}{2} K_2 t^{n-2} - \cdots + (-1)^n K_n.$$

In [5], Heinz proved that if $|K_1| \geq a < 0$ for the function $u = u(x, y)$ of two variables defined over $x^2 + y^2 < R^2$, then $R \leq 1/a$. This was generalized to n variables in Chern [1, Theorem 1] and independently in Flanders [4]. Again the hypothesis $|K_1| \geq a > 0$ leads to $R \leq 1/a$ and this is best possible.

Heinz [5] also considered a surface $u = u(x, y)$, $x^2 + y^2 < R^2$, for which the total (Gaussian) curvature

$$K_2 = \frac{rt - s^2}{w^4}, \qquad (w^2 = 1 + p^2 + p^2)$$

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