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A CLASS OF RIEMANNIAN METRICS ON A MANIFOLD

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0. Introduction

In his suggestive paper [3], R. Bott proved that if geodesics starting from a point p in a riemannian manifold M are all closed geodesics whose length of a lap is constant, then the number of conjugate points of p on a lap of these closed geodesics are constant, counting the multiplicity. This result has been extended recently by Nakagawa [9], who proved that if all geodesics starting from a point p with a constant length c come back to the point p (these are not necessarily closed geodesics), then the number of conjugate points on a lap of these closed geodesic segments are constant, counting the multiplicity.

If a stronger condition is assumed so that the cut point of p with respect to every geodesic starting from p may become a middle point of this closed geodesic segment, then the manifold M has a decomposition $M = D_p \cup {}_\varphi D_N$, as it is seen in Warner's paper [11], where D_p is a disk, N is a cut locus of p, which becomes a closed submanifold in this case, and D_N is a normal disk bundle of N in M.

In this paper, as an extension of these facts, it will be proved that *if a compact connected real analytic riemannian manifold M has a submanifold N such that the cut point of N with respect to every geodesic, which starts from N and whose initial direction is orthogonal to N has a constant distance π from N, then M has a decomposion $M = D_N \cup {}_\varphi D_{N'}$, where N' is the cut locus of N and D_N, $D_{N'}$ are normal disk bundles of N, N' respectively* (cf. Theorem 3.1). Of course, manifolds having such a decomposition are very special, but at any rate, it seems interesting to consider some details about that kind of manifold.

On a single manifold M, there are many, various riemannian metrics, which form a convex set. Each of these riemannian metrics, however, ought to be influenced by the topological structures of the manifold. Roughly speaking, one must be able to determine the topological structures of M by using only one riemannian metric, but at least at the present time it seems impossible. Therefore, it seems interesting to consider some useful class of riemannian metrics instead of a single metric or the whole metrics. In this paper, it

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