J. DIFFERENTIAL GEOMETRY
2 (1968) 197-202

REMARKS ON THE FIRST MAIN THEOREM IN EQUIDISTRIBUTION THEORY. I

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1. This is the first of a series commenting on the various aspects of the First Main Theorem (FMT) in several complex variables as proved by Chern [1] and Levine [4]. Our ultimate goal will be to recast the theorem of Chern [1, p. 537] in a form which can adequately "explain" the Fatou-Bieberbach example. We note that the FMT has recently been generalizd by Stoll [6].

We shall deal exclusively with the equi-dimensional case of the FMT, i.e. the situation where the dimensions of the domain and range manifolds are the same. In [4], Levine proved the nonintegrated FMT for a holomorphic $f\colon D \to P_pC$ by explicitly writing down a $(2p-1)$ form Λ in P_pC, and expressed the boundary integral $\int_{\partial D} f^*\Lambda$ as the difference of the counting function and the volume of the singular chain $f(D)$. The purpose of this short note is to point out that at least for the equi-dimensional case, which we are interested in, an a priori knowledge of Λ is unnecessary; a precise statement is given in the following theorem.

Let D be an orientable compact manifold of real dimension d, and M an orientable compact riemannian manifold without boundary also of dimension d. We adopt the convention throughout that Ψ denotes the volume form of M and that $\int_M \Psi = 1$. If $f\colon D \to M$ is C^∞, we write $v(D)$ for $\int_D f^*\Psi$ as usual.

Theorem. *For every $a \in M$, there exists an integrable $(d-1)$-form μ_a on M such that*:

(i) *If $f\colon D \to M$ is C^∞, and $f^{-1}(a)$ is finite and disjoint from ∂D, then*

$$v(D) = n(D, a) + \int_{\partial D} f^*\mu_a ,$$

where $n(D, a)$ denotes the algebraic number of points in $f^{-1}(a)$.

Communicated by J. Eells, Jr., November 13, 1967. Research partially supported by the National Science Foundation. The author is on sabbatical leave from the University of California, Berkeley, and is a Visiting Fellow at the Mathematical Institute of the University of Warwick. He wishes to thank Professors Zeeman and Fowler for their warm hospitality.