J. DIFFERENTIAL GEOMETRY
2 (1968) 185-190

ON THE GROUP OF CONFORMAL TRANSFORMATIONS OF A COMPACT RIEMANNIAN MANIFOLD. III

CHUAN-CHIH HSIUNG

1. Introduction

Let g_{ij}, R_{hijk}, $R_{ij} = R^k{}_{ijk}$ be respectively the metric, Riemann and Ricci tensors of a Riemannian manifold M^n of dimension n, and denote

$$(1.1) \qquad P = R^{hijk}R_{hijk}, \qquad Q = R^{ij}R_{ij} .$$

Throughout this paper all Latin indices take the values $1, \cdots, n$ unless stated otherwise, and repeated indices imply summation. In a recent paper [2] the author proved

Theorem 1. *Suppose that a compact Riemannian manifold $M^n (n > 2)$ with constant scalar curvature $R = g^{ij}R_{ij}$ admits an infinitesimal nonhomothetic conformal transformation v, and let L_v be the operator of the infinitesimal transformation v. If*

$$(1.2) \qquad a^2 L_v P + b(2a + nb) L_v Q = const.,$$

where a and b are constants such that

$$(1.3) \qquad c \equiv 4a^2 + 2(n-2)ab + n(n-2)b^2 > 0 ,$$

then M^n is isometric to a sphere.

In particular, when $a = 0$ or $b = 0$, Theorem 1 is reduced to a result of Yano [4], which is a generalization of some results of Lichnerowicz [3] and the author [1]. Yano pointed out that condition (1.3) is equivalent to that a and b are not both zero.

Very recently, Yano and Sawaki [5] obtained the following theorem similar to Theorem 1:

Theorem 2. *Suppose that a compact Riemannian manifold M^n $(n > 2)$ with constant R admits an infinitesimal nonhomothetic conformal transformation v. If*

$$(1.4) \qquad L_v L_v[(n-2)a^2 P + 4b(2a + b)Q] \leq 0 ,$$

Communicated November 28, 1967. Supported partially by the National Science Foundation grant GP-7513.