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RIEMANNIAN MANIFOLDS ADMITTING A CONFORMAL TRANSFORMATION GROUP

KENTARO YANO & SUMIO SAWAKI

1. Introduction

The purpose of the present paper is to generalize some of the known results on Riemannian manifolds with constant scalar curvature admitting a group of nonisometric conformal transformations.

Let M be a connected Riemannian manifold of dimension n, and g_{ji}, ∇_i, $K_{kji}{}^h$, $K_{ji} = K_{tji}{}^t$ and $K = K_{ji}g^{ji}$, respectively, the positive definite fundamental metric tensor, the operator of covariant differentiation with respect to the Levi-Civita connection, the curvature tensor, the Ricci tensor and the scalar curvature of M, where and in the sequel the indices $h, i, j, k, \cdots$ run over the range $1, \cdots, n$.

If we put

$$(1.1) \qquad G_{ji} = K_{ji} - \frac{K}{n} g_{ji} ,$$

$$(1.2) \qquad Z_{kji}{}^h = K_{kji}{}^h - \frac{K}{n(n-1)}(\delta_k^h g_{ji} - \delta_j^h g_{ki}) ,$$

we have

$$(1.3) \qquad Z_{tji}{}^t = G_{ji} , \quad G_{ji}g^{ji} = 0 .$$

When M admits an infinitesimal transformation v^h, we denote by $\mathscr{L}$ the operator of Lie derivation with respect to v^h. Thus, if M admits an infinitesimal conformal transformation v^h, we have

$$(1.4) \qquad \mathscr{L}g_{ji} = \nabla_j v_i + \nabla_i v_j = 2\rho g_{ji}, \quad \mathscr{L}g^{ih} = -2\rho g^{ih}$$

for a certain scalar field ρ. We denote the gradient of ρ by $\rho_i = \nabla_i \rho$.

For an infinitesimal conformal transformation v^h in M, we have [5]

$$(1.5) \qquad \mathscr{L}K_{kji}{}^h = -\delta_k^h \nabla_j \rho_i + \delta_j^h \nabla_k \rho_i - \nabla_k \rho^h g_{ji} + \nabla_j \rho^h g_{ki} ,$$

$$(1.6) \qquad \mathscr{L}K_{ji} = -(n-2)\nabla_j \rho_i - \Delta\rho g_{ji} ,$$

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