

A RESIDUE FORMULA FOR HOLOMORPHIC VECTOR-FIELDS

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1. Introduction

Let X be a holomorphic vector-field on the compact complex analytic manifold M . In an earlier note [3], the behavior of X near its zeros was related to the characteristic numbers of the tangent bundle to M , and the explicit form of this relation was computed in the most nondegenerate situation, that is, in the case of X vanishing at isolated points to the first order. Our aim here is to extend this result in two directions. On the one hand we consider the characteristic numbers of more general bundles E over M on which X "acts", and on the other hand we allow X to vanish along submanifolds of higher dimension but still only to the first order.

Both extensions are therefore essentially technical in nature. The first extension, to more general bundles, is especially direct and is worthwhile only in so far as it helps to clarify the arguments of [3]. The extension to higher dimensional zero-sets is less immediate and also seems to me of some interest for the following reason:

When X has isolated singularities the formulas in question may be derived from the generalized Lefschetz formula for transversal maps (see [2]). In the present more general case this is not so; one would first of all need a suitable generalization of the Lefschetz formula, and such an extension is available now only if X satisfies some additional restrictions, such as leaving a Riemannian structure invariant.¹ A Lefschetz formula for non-transversal maps is of course closely related to the Riemannian-Roch question, so that our ultimate motivation for this note is the hope that our results might be useful in an eventual purely geometric understanding of Riemann-Roch and its generalizations.

To describe our results we need to define two notions. First of all, by an

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¹ In recently completed forthcoming papers by M. Atiyah & I. Singer [1], on the one hand and Illusie [4] on the other, our formula is derived by quite different methods for vector fields of this type. Illusie also derives a mod p version of our theorems for transformations of order p .