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ALGEBRAIC DEFORMATION THEORY

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Introduction

Deformation theory dates back at least to Riemann's 1857 memoir on abelian functions in which he studied manifolds of complex dimension one and calculated the number of parameters (called moduli) upon which a deformation depends. Max Noether in his 1888 paper on the moduli of algebraic surfaces was apparently the first to consider deformations of manifolds of higher dimension. The modern theory of deformations of structures on manifolds was developed extensively in papers by Frölicher-Nijenhuis [2], Kodaira-Spencer [14], [15], Kodaira-Nirenberg-Spencer [13], and Spencer [22], [23]. The study of deformations of algebraic structures was initiated by Gerstenhaber, who, remarking that his methods extend to equationally defined algebraic structures, devoted his work [5] to consideration of associative algebras and graded and filtered rings.

Having the concept of deformation of algebraic structures (principally, associative algebras) and of analytic structures (principally, complex analytic manifolds), we are led to seek a deformation theory of mathematical structures in general. The present paper provides a step towards the development of a generalized deformation theory by introducing a type of cohomology, which we call "deformation cohomology," in the deformation theory of algebraic structures. The deformation cohomology is an algebraic analogue of the cohomology introduced by Haefliger [10] in the deformation theory of structures on manifolds. The latter is developed in greater detail in an unpublished communication from A. Douady to D. C. Spencer. The relationship between the deformation cohomology and the Hochschild cohomology, the latter reflecting the infinitesimal structure, is expressed by an exact, commutative diagram (see §§9-11).

A reasonable deformation theory for mathematical objects should incorporate the notion of a "deformation cohomology" which in turn is related to a cohomology reflecting the infinitesimal structure—the latter cohomology will be called, for simplicity, "infinitesimal cohomology."

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