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FIBRED SPACES WITH PROJECTABLE RIEMANNIAN METRIC

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Introduction

In our previous papers [8] and [9], we studied fibred spaces with invariant affine connection and those with invariant Riemannian metric, the fibres being 1-dimensional in both cases.

The idea of fibred spaces with invariant affine connection goes back to the representation of spaces with projective connection. To represent an n-dimensional manifold with projective connection, Princeton School used an $(n+1)$-dimensional manifold with affine connection admitting a concurrent vector field with respect to which the affine connection is invariant (See for example [5]), and Dutch School used a slightly general manifold with affine connection (See for example, [4]). They all identified a point in the manifold with projective connection with a trajectory of the vector field with respect to which the affine connection is invariant.

The idea of fibred spaces with invariant Riemannian metric goes back to the five dimensional Riemannian space considered by Th. Kaluza [1] and O. Klein [2] for getting a unified field theory of gravitation and electromagnetism. To represent the space-time, they used a 5-dimensional Riemannian space admitting a unit vector field with respect to which the Riemannian metric is invariant, and identified a point in the space-time with a trajectory of the unit vector field with respect to which the 5-dimensional Riemannian metric is invariant.

In the present paper, we study fibred spaces with Riemannian metric under the assumption that the Riemannian metric is projectable instead of being invariant (See [3], [7]). In §1, we state definitions and study some properties of a fibred space with projectable Riemannian metric, and in §2 we develop the tensor calculus in the space. §3 is devoted to the discussions on the Riemannian connection and the induced connection. We discuss geodesics in §4, and structure equations and curvatures in §5. In the last §6, we assume that the Riemannian metric is *invariant* with respect to a not necessarily unit vector field tangent to the fibre, and the manifold is then slightly more general than that we studied in [9].

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