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EINSTEIN HYPERSURFACES IN A KÄHLERIAN MANIFOLD OF CONSTANT HOLOMORPHIC CURVATURE

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Introduction

In his dissertation Brian Smyth studied the complete hypersurfaces in a complex space-form whose induced metric is einsteinian and proved that these are either totally geodesic or certain hyperquadrics of the complex projective space. We wish to show in this note that the corresponding *local* theorem is true:

Theorem. *Let* V *be a kählerian manifold of dimension* ≥ 3 *with constant holomorphic sectional curvature* K. *Let* f: M $\to$ V *be a holomorphically immersed hypersurface such that the induced metric is einsteinian. Then, if* K ≤ 0, M *is totally geodesic. If* K > 0 *and* V *is identified with the complex projective space,* M *is either totally geodesic or a hypersphere (cf.* §3 *for definition).*

1. Preliminaries on kählerian geometry

We will summarize the basic formulas of kählerian geometry. For details cf. [1].

In order to avoid repetitions it will be agreed that our indices have the following ranges throughout this paper:

$$
\begin{aligned}
&1 \leq i, j, k, l \leq n,\\
&1 \leq \alpha, \beta, \gamma, \delta \leq n+1,\\
&0 \leq A, B, C, D \leq n+1.
\end{aligned}
\tag{1}
$$

Let V be a kählerian manifold of complex dimension $n+1$. The metric defines an hermitian scalar product in the tangent spaces of V and a connection of type (1, 0) under whose parallelism the scalar product is preserved. More precisely, let $e_\alpha(x)$ be a field of unitary frames, defined for x in a neighborhood of V. Its dual coframe field consists of $n+1$ complex-valued linear differential forms θ_α of type (1, 0) such that the hermitian metric can be written

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