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CHARACTERS OF SL(2) REPRESENTATIONS OF GROUPS

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Abstract

Given a compact orientable surface Σ, let $S(\Sigma)$ be the set of isotopy classes of essential unoriented simple loops in the surface. We determine a complete set of relations for a function defined on $S(\Sigma)$ to a field K to be the character of an $SL(2, K)$ representations. Furthermore, the relations are supported in the 1-holed torus and the 4-holed sphere subsurfaces. This establishes that Grothendieck's reconstruction principle is valid for $SL(2, K)$-character varieties of surface groups. As a consequence, we obtain an explicit description of the set of all characters of $SL(2, K)$ representations of a group.

1. Introduction

1.1. Given a field K and a representation ρ of a group to $SL(2, K)$, the *character* of the representation sends a group element g to the trace of the matrix $\rho(g)$. One of the result of the paper is the following,

Theorem. *Suppose K is a field so that each quadratic equation with coefficients in K has a root in K. Then a K-valued function defined on a group is the character of a $SL(2, K)$ representation of the group if and only if its restriction to each 2-generator subgroup is a $SL(2, K)$ character.*

The $SL(2, K)$ characters on 2-generator groups are well understood since the work of Fricke-Klein [7] and Vogt [33]. They are governed by the trace identity: $tr(AB) + tr(A^{-1}B) = tr(A)tr(B)$ for $SL(2, K)$ matrices A, B. In [14], Helling gave an elegant axiomatic approach to characters based on the above trace identity. Following Helling, a K-valued function f defined on a group G is called a *K-trace function* if (1) $f(xy) + f(x^{-1}y) = f(x)f(y)$ for all x, y in G and (2) $f(id) = 2$ where

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