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GREEN FUNCTIONS AND CONFORMAL GEOMETRY

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Abstract

We use the Green function of the Yamabe operator (conformal Laplacian) to construct a canonical metric on each locally conformally flat manifold different from the standard sphere that supports a Riemannian metric of positive scalar curvature. In dimension 3, the assumption of local conformal flatness is not needed. The construction depends on the positive mass theorem of Schoen-Yau. The resulting metric is different from those obtained earlier by other methods. In particular, it is smooth and distance nondecreasing under conformal maps. We analyze the behavior of our metric if the scalar curvature tends to 0. We demonstrate that the canonical metrics converge under surgery-type degenerations to the corresponding metric on the limit space. As a consequence, the L^2−metric on the moduli space of scalar positive locally conformally flat structures is not complete. The example of $S^1 \times S^2$ as underlying manifold is studied in detail.

Introduction

For the sake of simplicity, we assume throughout this introduction that all occurring manifolds are compact.

In understanding spaces of complex structures, it has proved to be useful to construct "canonical" metrics on complex manifolds. Such a "canonical" metric ideally is uniquely determined by the underlying complex structure, depends smoothly on that structure, and has an analyzable behavior as the underlying structure degenerates in some explicit manner. Such a metric on each complex structure then gives rise to a metric on the corresponding moduli space[1] by taking the L^2-product of tangent vectors to the moduli space — which can be expressed as harmonic sections of a certain bundle on the underlying complex manifold — w.r.t. the canonical metric.

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[1]Leaving aside the issue of smoothness of that space