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THE SYMPLECTIC GLOBAL COORDINATES ON THE MODULI SPACE OF REAL PROJECTIVE STRUCTURES

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A *convex* real projective structure on a smooth surface M is a representation of M as a quotient of a convex domain $\Omega \subset \mathbb{RP}^2$ by a discrete group $\Gamma \subset \mathbf{PGL}(3, \mathbb{R})$ acting properly and freely on Ω. If $\chi(M) < 0$, then the equivalence classes of convex real projective structures form a moduli space $\mathfrak{P}(M)$ which is an extension of the Teichmüller space $\mathfrak{T}(M)$.

Wolpert [17] proved that the Weil-Petersson Kähler form of the Teichmüller space $\mathfrak{T}(M)$ of a closed surface $\Sigma(g, 0)$ with $\chi(M) < 0$ is expressed by

$$\omega = \sum_{i=1}^{g} d\ell_i \wedge d\theta_i,$$

where ℓ_i, θ_i are Fenchel-Nielsen coordinates on $\mathfrak{T}(M)$. In this paper, I will prove $\mathfrak{P}(M)$ has analogous properties.

In Section 1, we study the set of positive hyperbolic elements $\mathbf{Hyp}_+$ of $\mathbf{PGL}(3, \mathbb{R})$ since the holonomy group Γ of a convex real projective structure lies in $\mathbf{Hyp}_+$. In Section 2, we show the parameters (ℓ, m) on $\mathfrak{P}(M)$ extend Fenchel-Nielsen's length parameter ℓ. Let π be the fundamental group of M and G a connected algebraic Lie group. In Section 3, we study local properties of $\mathrm{Hom}(\pi, G)/G$ since $\mathfrak{P}(M)$ embeds onto an open subset of $\mathrm{Hom}(\pi, \mathbf{PGL}(3, \mathbb{R}))/\mathbf{PGL}(3, \mathbb{R})$. In Section 4,

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