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FUKAYA-FLOER HOMOLOGY OF $\Sigma \times \mathbb{S}^1$ AND APPLICATIONS

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Abstract

We determine the Fukaya-Floer (co)homology groups of the three-manifold $Y = \Sigma \times \mathbb{S}^1$, where Σ is a Riemann surface of genus $g \geq 1$. These are of two kinds. For the 1-cycle $\mathbb{S}^1 \subset Y$, we compute the Fukaya-Floer cohomology $HFF^*(Y, \mathbb{S}^1)$ and its ring structure, which is a sort of deformation of the Floer cohomology $HF^*(Y)$. On the other hand, for 1-cycles $\delta \subset \Sigma \subset Y$, we determine the Fukaya-Floer homology $HFF_*(Y, \delta)$ and its $HF^*(Y)$-module structure.

We give the following applications:

- We show that every four-manifold with $b^+ > 1$ is of finite type.
- Four-manifolds which arise as connected sums along surfaces of four-manifolds with $b_1 = 0$ are of simple type and we give constraints on their basic classes.
- We find the invariants of the product of two Riemann surfaces both of genus greater than or equal to one.

1. Introduction

The structure of Donaldson invariants of 4-manifolds has been found out by Kronheimer and Mrowka [16] and Fintushel and Stern [8] for a large class of 4-manifolds (those of simple type with $b_1 = 0$, $b^+ > 1$) making use of universal relations coming from embedded surfaces. In order to analyse general 4-manifolds, we need to set up first the right framework for getting enough universal relations. It is the purpose of

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