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# A NEW CONSTRUCTION OF COMPACT 8-MANIFOLDS WITH HOLONOMY Spin(7)

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## 1. Introduction

In Berger's classification [1] of holonomy groups of Riemannian manifolds there are two special cases, the exceptional holonomy groups $G_2$ in 7 dimensions and Spin(7) in 8 dimensions. Bryant [2] and Bryant and Salamon [3] showed that such metrics exist locally, and wrote down explicit, complete metrics with holonomy $G_2$ and Spin(7) on noncompact manifolds.

The first examples of metrics with holonomy $G_2$ and Spin(7) on *compact* 7- and 8-manifolds were constructed by the author in [10], [11], [12]. The survey paper [13] provides a good introduction to these constructions. Here is a brief description of the method used in [10] to construct compact 8-manifolds with holonomy Spin(7), divided into four steps.

(a) We start with a flat Spin(7)-structure $(\Omega_0, g_0)$ on the 8-torus $T^8$, and a finite group $\Gamma$ of isometries of $T^8$ preserving $(\Omega_0, g_0)$. Then $T^8/\Gamma$ is an *orbifold*, a singular manifold with only quotient singularities.

(b) For certain $\Gamma$ one can resolve the singularities of $T^8/\Gamma$ in a natural way, using complex geometry. This gives a nonsingular, compact 8-manifold $M$, and a projection $\pi : M \to T^8/\Gamma$.

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